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Jeudi, le 06 Octobre 2011, à 14:30

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## **When is a multiplicative function small on average ?**

Let  $f$  be a multiplicative function. The main problem we will be concerned with in this talk is understanding when  $f$  is small on average. Halász showed that, unless  $f$  ‘pretends to be’  $n^{it}$  for some small  $t$ , this is true and gave quantitative estimates on the rate of decay of the partial sums of  $f$ . The estimate provided by Halász’s theorem is in general tight but there are functions  $f$  for which it is far from the truth. A natural question that arises is to classify the functions  $f$  whose partial sums are significantly smaller than what one might predict by Halász’s theorem. More precisely, if  $\sum_{n \leq x} f(n) \ll x(\log x)^{-A}$  for some big  $A$ , then what can we say about  $f$ ? If  $f$  is real valued, we show that either  $\sum_{p \leq x} (1 + f(p)) \ll x(\log x)^{-A/2+O(1)}$  or  $\sum_{p \leq x} f(p) \ll x(\log x)^{-A/2+O(1)}$ , depending on whether the Dirichlet series corresponding to  $f$  vanishes at the point 1 or not. In other words,  $f$  looks very much like the Mobius function or its prime values are small on average. We also give an analogous result for the general case of a complex valued  $f$ . Finally, we show how these methods can be used to give a new proof of the prime number theorem in arithmetic progressions.