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Born on April 2, 1980
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Positions

2007–2009 **Teaching and research assistant (*A.T.E.R.*)** University of Nancy I (Nancy, France).

2007–2008 **Pre-Doc** University of Cologne (Cologne, Germany) *from september to january under the funding of LieGrits European network.*

2004–2007 **Teaching and research assistant (*Allocataire Moniteur*)** University of Nancy I (Nancy, France).

Education

2004–2008 **Ph.D Mathematics (*Doctorat de mathématiques*),** *title :"Some properties of representations of the Lie superalgebra* $\mathfrak{gl}(m,n)$*", under the supervision of Caroline* Gruson*, soutenu le 4 décembre 2008 à l'université Nancy I, Presented to Michel* Duflo*, Caroline* Gruson*, Bernhard* Keller*, Peter* Littelmann*, Guy* Rousseau *(président), Vera* Serganova*, University of Nancy I (Nancy, France).*

2004 **M.Sc. Mathematics (*D.E.A. de Mathématiques*),** *with honours (year 2), the master's thesis focused on "Kazhdan-Lusztig polynomials" (based Wolfgang Soergel's article "Kazhdan-Lusztig polynomials and a combinatoric for tilting modules"),* University of Nancy I (Nancy, France).

2003 **M.Sc. Mathematics (*Maîtrise de Mathématiques*),** *with honours (year 1), the master's thesis focused on "Representation theory" (based on William Fulton et Joe Harris : "Representation Theory : a first course"),* University of Nancy I (Nancy, France).

2002 **B.Sc. Mathematics (*Licence de Mathématiques* and *DEUG MIAS*),** *with honours* University of Nancy I (Nancy, France).

1998 **A level (maths physcics sciences) (High School end Diploma majoring in science, french *Bacalauréat S*)** *with honours* Lycée Jeanne d'Arc (Nancy, France).

Workshop

2009-2010	**Workshop on Hecke algebras** (Nancy, France).
2008-2009	**Workshop on the saturation theorem of Kapovich and Millson** (Nancy, France).
2007-2008	**Workshop on Lie superalgebra** (Nancy, France).
	Workshop on deformations of algebraic schemes (Cologne, Germany).
2006-2007	**Workshop on Mirković-Vilonen's cycle** (Nancy, France).
	Workshop on Buildings (Nancy, France).
2005-2006	**Workshop on the article "On associated Variety for Lie Superalgebras" (M.Duflo, V.Serganova, arXiv RT/0507198)** (Nancy, France).
	Workshop on Littelmann path model (Nancy, France).
2004-2005	**Workshop on character formula for the Lie superalgebras** $\mathfrak{gl}(m,n)$ (Nancy, France).

Presentations and scientific meetings

2009	**Workshop "Modern geometry"** Luxembourg (Luxembourg) *(23 Nov., talk "Finite dimensional representations of the strange Lie superalgebra* $\mathfrak{p}(3)$*").*
	General Mathematics Seminar Luxembourg (Luxembourg) *(9 June, talk "Crystal bases for simple modules of the quantum group* $U_q(\mathfrak{gl}(m,n))$*").*
	Representation theory seminar (*Colloque Tournant de théorie des représentations*) Paris (France) *(13 Feb., talk : "Bases cristallines des modules simples du groupe quantique* $U_q(\mathfrak{gl}(m,n))$*").*
2008	**Number theory and algebra seminar** Besançon (France) *(11 Dec., talk : "Bases cristallines des modules simples du groupe quantique* $U_q(\mathfrak{gl}(m,n))$*").*
	Nancy-Metz-Strasbourg-Reims representation theory and harmonic analysis seminar Strasbourg (France) *(27 and 28 Nov., talk : "Base cristalline des modules simples du groupe quantique* $U_q(\mathfrak{gl}(m,n))$*").*
	Algebra seminar Cologne (Germany) *(1 Jul., talk "Crystal bases for simple modules of the quantum group* $U_q(\mathfrak{gl}(m,n))$*").*
	Bonn-Köln algebra seminar Bonn (Germany) *(29 Jan.).*
2007	**Algebra seminar** Cologne (Germany) *(23 Oct., talk "The Lie superalgebra* $\mathfrak{gl}(2,2)$*).*
	Lie group and harmonic analysis seminar Nancy (France) *(30 May, talk : "Sur la super-algèbre de Lie* $\mathfrak{gl}(2,2)$*).*
	Representation theory seminar (*Colloque Tournant de théorie des représentations*) Amiens *(13-15 Jan.).*
2006	**Geometry and representation seminar** CIRM *(4-8 Dec.).*
	Winterschool "Representation Theory and Applications" Oberwalfach (Germany) *(19-25 Nov.).*
	Representation theory seminar (*Colloque Tournant de théorie des représentations*) Nancy (France) *(13-15 Jan.).*
2005	**Geometry and representation seminar** CIRM (France) *(11-15 Apr.).*
	Representation theory seminar (*Colloque Tournant de théorie des représentations*) Grenoble (France) *(13-15 Jan.).*
2004	**Summerschool 'Quivers, representation theory and geometry** CIRM (France) *(24-28 May).*
	Representation theory seminar (*Colloque Tournant de théorie des représentations*) Montpellier (France) *(du 15 au 17 janvier).*

Teaching activities

2008–2009	**Preparation, French admission teacher test (*CAPES interne de mathématiques*,** University of Nancy I (France) : *Analysis, algebra and probability.*
	Lectures and Exercise in analysis to first year B.Sc. students in Mathematics and Physics(*LMI-SM L1S2*), University of Nancy I (France).
	Lectures and Exercise in geometry to second year B.Sc. students in Mathematics(*LMI L2S2*), University of Nancy I (France).
2007–2008	**Lectures and Exercise in mathematics to first year B.Sc. students in Mathematics, Automatism and Computer Science(*LMI-EEA L1S1*),** University of Nancy I (France).
2004–2007	**Exercise in Mathematics to second year engineer students,** ESSTIN (France).
2001-2003	**Tutorials in Mathematics to first year B.Sc. students (*DEUG MIAS*),** University of Nancy I (France).

Papers

F. Drouot *Quelques Propriétés des Représentations de la Super-algèbre de Lie* $\mathfrak{gl}(m,n)$, PhD thesis, Université Henri Poincaré, Nancy, 2008.

In preparation :

- Character formula for simple $\mathfrak{gl}(2,2)$-modules of finite dimension,
- Crystal bases in $U_q(\mathfrak{gl}(m,n))$,
- Finite dimensional module of the strange Lie superalgebra $\mathfrak{p}(3)$.

Research summary and project

My main topic is the category $\mathcal{F}$ of finite dimensional representations of simple Lie superalgebras (in particular $\mathfrak{gl}(m,n)$ and $\mathfrak{p}(n)$) on which a Cartan subalgebra $\mathfrak{h}$ acts diagonally. It is a highest weight modules category. As opposed to what happens in the case of reductive Lie algebras, this category is not semi-simple in general.

I (want to) study three things in this category :
- the character formula for simple modules (as in the case of Lie algebras, a module which is a semi-simple $\mathfrak{h}$-module is completely determined by the action of $\mathfrak{h}$, hence by its character). Recently Gruson and Serganova (in [7]) proved a character formula for the orthosymplectic and the general linear Lie superalgebras. A character formula was already given by Serganova for the general linear Lie superalgebra (in [11]) and by Serganova and Penkov for $\mathfrak{q}(n)$ (in [12]), later by Brundan with a completely different approach (in [2] and [3]). It is still an open problem for $\mathfrak{p}(n)$. Here the notations $\mathfrak{q}(n)$ and $\mathfrak{p}(n)$ refer to Kac's classification (in [9]).
- the structure of $\mathcal{F}$. We know, according to Gabriel, that the knowledge of the full subcategory of projective objects in $\mathcal{F}$ is sufficient to reconstruct $\mathcal{F}$. We need therefore the structure of indecomposable projective modules and morphisms between them. Recently Brundan and Stroppel have given the answer for the general linear Lie superalgebra (in [4]). Gruson has done it for $\mathfrak{osp}(3,2)$ (in [6]).
- decompose tensor products of simple modules. In the case of reductive Lie algebras, crystal bases are an important tool for this question. Except in some special cases (e.g. tensor product with the standard representation for $\mathfrak{gl}(m,n)$) it is still an open problem.

Maximally atypical block of $\mathfrak{gl}(m,n)$

Let $\mathfrak{g}$ be the Lie superalgebra $\mathfrak{gl}(m,n)$ over $\mathbb{C}$. A simple module which is a direct factor as soon as it appears as subquotient is called typical; typical modules are well known. Non-typical modules (they are also called atypical) are smaller than what is expected, we associate to these modules an integer called the degree of atypicality that measures, in this case, the lack of size. Serganova proved that in the block decomposition of the category $\mathcal{F}$, modules in the same block have the same atypicality degree (so one can define the degree of atypicality of a block). She proved also that one has an equivalence of categories between a k-atypical block of $\mathcal{F}$ and the block, of the category $\tilde{\mathcal{F}}$ of $\mathfrak{gl}(k,k)$, containing the trivial module.

In 1996, in [11], Vera Serganova has determined a character formula for all simple $\mathfrak{gl}(m,n)$-modules of finite dimension, but this character formula is an infinite sum, and it is therefore difficult to see whether a given weight appears in a module.

One way to get a character formula of finite length is to decompose simple $\mathfrak{gl}(m,n)$-modules as a direct sum of simple $(\mathfrak{gl}_m(\mathbb{C}) \times \mathfrak{gl}_n(\mathbb{C}))$-modules. The combinatorics of weights of $\mathfrak{gl}(2,2)$ being quite simple, one can decompose simple finite-dimensional $\mathfrak{gl}(2,2)$-modules in direct sum of simple modules of the even part $\mathfrak{g}_{\underline{0}}$ of $\mathfrak{g}$ (which is $\mathfrak{gl}_2(\mathbb{C}) \times \mathfrak{gl}_2(\mathbb{C})$). One may notice that a typical generic module (resp. 1-atypical, resp. 2- atypical) has 16 (resp. 8, resp. 4) $\mathfrak{g}_{\underline{0}}$-components.

Typical blocks (resp. 1-atypical blocks) are described by Kac in [8] (resp. Germoni in [5]), therefore to understand the category $\mathcal{F}$ of $\mathfrak{gl}(2,2)$ it is enough to understand the 2-atypical block $\mathcal{F}_0$ (it is the one containing the trivial representation).

I have computed the extension groups between simple objects of $\mathcal{F}_0$, this gave me the Loewy filtration of the indecomposable projective modules, and therefore the morphisms between two non-isomorphic indecomposable projective modules.

I have also proved that the algebra of $\mathfrak{gl}(m,n)$-linear endomorphisms of an indecomposable projective module is self injective.

Using the techniques of my Ph.D. thesis I have proved that the algebra of endomorphisms of a 2-atypical indecomposable projective module is isomorphic to $\mathbb{C}[X,Y]/(X^2,Y^2)$ and also computed the morphisms between projective modules. One can notice that these results are special cases of the results of Brundan and Stroppel (in [4]).

Crystal bases for representations of the quantum group associate to $\mathfrak{gl}(m,n)$

Crystal bases are powerful combinatoric tools which help to understand the structure of representations of quantum deformations of the enveloping algebras of a semi-simple Lie algebra (and the enveloping algebra itself). In 2000, Benkart, Kang and Kashiwara in [1], constructed a quantum group associated to the Lie superalgebra $\mathfrak{gl}(m,n)$; they also constructed crystal bases for the objects of the semi-simple category $\mathcal{O}_{BKK}$ of tensor powers of the standard representation $\mathbf{V}$ of $\mathfrak{gl}(m,n)$.

In the case of the Lie algebra $\mathfrak{gl}_{m+n}(\mathbb{C})$, every finite dimensional simple module can be obtained as a subquotient of a suitable tensor power of $\mathbf{V}$. This is no longer true for $\mathfrak{gl}(m,n)$, where we have to consider tensor powers of $\mathbf{V} \oplus \mathbf{V}^\star$.

I weakened the definition of crystal basis in order to show that if M (resp. N) is in $\mathcal{O}_{BKK}$ (resp. its analogue using $\mathbf{V}^\star$ instead of $\mathbf{V}$) then the module $M \otimes N$ has a crystal basis. All the simple modules appear as subquotients of such a module. However we have to be careful, since it is tricky to obtain a crystal basis for a subquotient. Besides, for $U_q(\mathfrak{gl}(2,2))$, it's possible to check that we have a correspondence between connected components of the crystal graph associated to the crystal basis and the simple subquotients, and so we know how to get crystal bases for simple modules in this case.

In the case $m=n$, the connected components of the crystal graph associated to the crystal basis of $\mathbf{V} \otimes \mathbf{V}^\star$ are crystal bases of some subquotients of this module. I would like to prove that it still holds for modules of the form $M \otimes N$ (as described above). In order to prove this result (which would produce crystal bases for all simple modules) it is necessary to get additional information, for example the knowledge of the dimensions of simple modules.

Finite dimensional representations of the strange Lie superalgebra $\tilde{\mathfrak{p}}(n)$

Let $\mathfrak{g}$ be the strange Lie superalgebra $\tilde{\mathfrak{p}}(n)$ over $\mathbb{C}$. The even part $\mathfrak{g}_{\underline{0}}$ is the reductive Lie algebra $\mathfrak{gl}_n(\mathbb{C})$. Le E be the standard module of $\mathfrak{g}_{\underline{0}}$, the odd part is isomorphic to $\Lambda^2 E \oplus \mathrm{S}^2 E$. One can see that $\tilde{\mathfrak{p}}(n)$ is the Lie algebra of all endomorphisms of a (n,n)-dimensional vector superspace which preserve a given odd non-degenerate symmetric form.

We can define a root system for $\tilde{\mathfrak{p}}(n)$ and separate the roots into tow sets : positive ones and negatives ones, but the cardinal of those two sets is different. For this Lie superalgebra there is no Berstein-Gelfand-Gelfand reciprocity theorem.

The center of the universal enveloping algebra $\mathcal{U}$ of $\mathfrak{g}$ is trivial, so we can't define a central character. In 2002, in [10], Vera Serganova proved that the quotient $\overline{\mathcal{U}}$ of the universal enveloping algebra of $\tilde{\mathfrak{p}}(n)$ by its radical has a non-trivial center. She also proved that the character of this center separates typical finite dimensional simple representations. A finite dimensional simple representation is called typical if it is isomorphic to a so called Kac module (which is an induction to $\mathfrak{g}$ of a simple $\mathfrak{g}_{\underline{0}}$-module trivially extended on $\mathfrak{g}_{\underline{0}} \oplus \mathrm{S}^2 E$). She proved that if we study simple representations of $\overline{\mathcal{U}}$ instead of $\mathcal{U}$, we gain some additional information. She also gave a character formula for typical representations.

For $\tilde{\mathfrak{p}}(3)$, studying filtrations of Kac modules by simple $\mathfrak{g}_{\underline{0}}$-modules one can deduce a filtration of these modules by simple $\mathfrak{g}$-modules, this allows us to write a character formula for all finite dimensional simple modules (not only for the typical ones but also for the non-typical ones).

There are two topics I would like to study. The first one is to understand the category of finite dimensional representations of the Lie superalgebra $\tilde{\mathfrak{p}}(3)$, it will be done by computing the structure of indecomposable projective modules and by studying morphisms between them. The second one is to get a character formula for finite dimensional simple modules of $\tilde{\mathfrak{p}}(n)$.

[1] G. Benkart, S.-J. Kang and M. Kashiwara *Crystal bases for the quantum superalgebra* $U_q(\mathfrak{gl}(m,n))$, J. Amer. Math. Soc. , Volume 13 No 2 pp 295–311, 2000.

[2] J. Brundan *Kazhdan-Lusztig polynomials and character formulae for the Lie superalgebra* $\mathfrak{gl}((m|n)$, J. Amer. Math. Soc. , Volume 16 No 1 pp 185–231, 2003.

[3] J. Brundan *Kazhdan-Lusztig polynomials and character formulae for the Lie superalgebra* $\mathfrak{q}(n)$, Adv. Math., Volume 182 No 1 pp 28–77, 2004.

[4] J. Brundan and C. Stroppel *Highest weight categories arising from Khovanov's diagram algebra IV : the general linear supergroup* , arXiv, 0907.2543 math.RT, 2009.

[5] J. Germoni *Représentations indécomposables des superalgèbres de Lie spéciales linéaires*, Ph.D. thesis, Université Louis Pasteur Strasbourg, 1997.

[6] C. Gruson *Cohomologie des modules de dimension finie sur la super algèbre de Lie* $\mathfrak{osp}(3,2)$, J. Algebra, Volume 259 No 2 pp 581–598, 2003.

[7] C. Gruson and V.Serganova *Cohomology of generalized supergrassmannians and character formulae for basic classical Lie superalgebras* , arXiv, 0906.0918 math.RT, 2009.

[8] V. Kac *Characters of typical representations of classical Lie superalgebras*, Comm. Algebra, Volume 5 No 8 pp 889–897, 1977.

[9] V. Kac *Lie superalgebras*, Advances in Math., Volume 26 No 1 pp 8–96, 1977.

[10] V. Serganova *On representations of the Lie superalgebra P(n)*, J. Algebra, Volume 258 No 2 pp 615–630, 2002.

[11] V. Serganova *Kazhdan-Lusztig Polynomials and Character Formula for the Lie superalgebra* $\mathfrak{gl}(m,n)$, Selecta Math., Volume 2 No 4 pp 607–651, 1996.

[12] V. Serganova and I.Penkov *Characters of irreducible G-modules and cohomology of G/P for the Lie supergroup* $G = Q(N)$, J. Math. Sci., Volume 84 No 5 pp 1382–1412, 1997.